

Component 02

Algorithms and programming

Sorting Algorithms and Searching Algorithms

© Matthew Robinson

SORTING ALGORITHMS

BUBBLE SORT

Bubble sort is a brute force and iterative sorting algorithm where each adjacent item in the array is compared. If the item on the right is less than the item on the left they are swapped. The last element of the array will be in the correct place.

Process

- 1) Set swapMade variable to True.
- 2) While swapMade is True, make passes through the list:
 - for all items list.LENGTH-2 (all but the last item):
 - compare the first and the second item in the list:
 - if the first item is greater than the second item, then they are not in order – swap them and set swapMade variable to True; or
 - if the first item is less than the second item, then they are in order; repeat the comparison process until the end of the list is reached;
 - when the end of the list is reached, begin another pass.
- 3) On the final pass, all of the items should be sorted and therefore swapMade will remain False and the algorithm will finish.

Pseudocode

```
FUNCTION bubbleSort(list)
  swapMade = True
  WHILE swapMade == True
    swapMade = False
    FOR i=0 TO list.LENGTH - 2 // iterate through all but the last item
      IF list[i] > list[i+1] THEN
        temp = list[i+1]
        list[i+1] = list[i]
        list[i] = temp
        swapMade = True
      ENDIF
    NEXT i
  ENDWHILE
  RETURN list
ENDFUNCTION
```

The algorithm can be improved by decrementing the number of items to be inspected on each pass, as these missed items are assumed to be sorted.

Process

- 1) Set swapMade variable to True.
- 2) Set passes variable to list.LENGTH-2 (all but the last item)
- 3) While swapMade is True, make passes through the list:
 - for all items, but the last item, in the list:
 - compare the first and the second item in the list:
 1. if the first item is greater than the second item, then they are not in order – swap them and set swapMade variable to True; or
 2. if the first item is less than the second item, then they are in order; repeat the comparison process until the end of the list is reached;
 - when the end of the list is reached, begin another pass;
 - decrement the passes variable.
- 4) On the final pass, all of the items should be sorted and therefore swapMade will remain False and the algorithm will finish.

Pseudocode

```
FUNCTION bubbleSort(list)
  swapMade = True
  passes = list.LENGTH-2 // iterate through all but the last item
  WHILE swapMade == True
    swapMade = False
    FOR i=0 TO passes
      IF list[i] > list[i+1] THEN
        temp = list[i+1]
        list[i+1] = list[i]
        list[i] = temp
        swapMade = True
      ENDIF
    NEXT i
    passes = passes - 1
  ENDWHILE
  RETURN list
ENDFUNCTION
```

This improvement means that unnecessary passes are not made as the items at the end of the list do not need to be compared since they are sorted.

SORTING ALGORITHMS

INSERTION SORT

Insertion sort is an iterative sorting algorithm that works by dividing a list into two parts: a sorted portion; and an unsorted portion. The elements in the list are inserted, one at a time, into their correct position in the sorted portion.

Process

- 1) Make the first item in the list the sorted portion of the list and the remaining items are the unsorted portion of the list.
- 2) While there are items in the unsorted list:
 - take the first item in the unsorted list;
 - while there is an item to the left of the first item in the unsorted list:
 - swap with that item;
 - the sorted list is now one item bigger, as the first item of the unsorted list has become a member of the sorted list.

Pseudocode

```
FUNCTION insertionSort(list)
  FOR i=0 TO list.LENGTH-1 // iterate through all but the last item
    currentItem = list[i] // take the first item in the unsorted list
    position = i // set the position to the current index value
    // while there are items in the unsorted list
    WHILE position > 0 AND list[position - 1] > currentItem
      // swap the items
      list[position] = list[position - 1]
      position = position - 1
    ENDWHILE
    list[position] = currentItem
  NEXT i
  RETURN list
ENDFUNCTION
```

SORTING ALGORITHMS

MERGE SORT

Merge sort is a divide and conquer sorting algorithm where the list recursively partitioned in to halves, until each sublist is of length one, and therefore sorted by definition as the single item is the smallest and the largest in that sublist. The sublists are then sorted and merged into larger sublists until they are recombined into a single sorted list. The implementation of a merge sort is usually recursive as the way it solves the problem is inherently recursive so it naturally lends itself to this style of implementation.

Process

- 1) The list is split into two halves.
- 2) For the left half:
 - split the left half into halves until each sublist is of length one.
 - merge the pair of sublists on the left half by repeating this process until all items are in the merged list:
 - comparing the first item in the left half with the first item in the right half;
 - if the item in the left half is less than the item in the right half, add the item from the left half to the merged list and read the next item from the left half;
 - if the item in the right half is less than the item in the left half, add the item from the right half to the merged list and read the next item from the right half;
 - once either list is empty, any remaining items are added to the merged list.
- 3) For the right half:
 - split the right half into halves until each sublist is of length one.
 - merge the pair of sublists on the right half by repeating this process until all items are in the merged list:
 - comparing the first item in the left half with the first item in the right half;
 - if the item in the left half is less than the item in the right half, add the item from the left half to the merged list and read the next item from the left half;
 - if the item in the right half is less than the item in the left half, add the item from the right half to the merged list and read the next item from the right half;
 - once either list is empty, any remaining items are added to the merged list.
- 4) Merge the left half and the right half by repeating this process until all items are in the merged list:
 - comparing the first item in the left half with the first item in the right half;
 - if the item in the left half is less than the item in the right half, add the item from the left half to the merged list and read the next item from the left half;
 - if the item in the right half is less than the item in the left half, add the item from the right half to the merged list and read the next item from the right half;
 - once either list is empty, any remaining items are added to the merged list.

SORTING ALGORITHMS

MERGE SORT

Pseudocode

```
PROCEDURE mergeSort(list)
  // base case
  IF list.LENGTH > 1 THEN // if list is not, by definition, sorted
    mid = list.LENGTH DIV 2 // performs integer division to find
                           // the midpoint of the list
    leftHalf = mergeList[:mid] // left half of list
    rightHalf = mergeList[mid:] // right half of list

    // recursive case
    mergeSort(leftHalf) // recursive call for leftHalf
    mergeSort(rightHalf) // recursive call for rightHalf

    i = 0 // pointer to item in leftHalf (starting at the first item)
    j = 0 // pointer to item in rightHalf (starting at the first item)
    k = 0 // pointer to item in list (starting at the first item)

    // while the first item in the leftHalf is less than the length of the
    // length of the leftHalf AND the first item in the rightHalf is less
    // than the length of the rightHalf

    i.e. while there are still item in the leftHalf and rightHalf of the
    sublists
    WHILE i < leftHalf.LENGTH AND j < rightHalf.LENGTH
      // if the item at the pointer in the leftHalf is less than the item
      // at the pointer in the rightHalf
      IF leftHalf[i] < rightHalf[j] THEN
        // the item at the pointer in leftHalf the becomes is added to
        // the list
        list[k] = leftHalf[i]
        // increment the pointer pointing to the item in the leftHalf
        i = i + 1
      ELSE
        // the item at the pointer in rightHalf the becomes is added to
        // the list
        list[j] = rightHalf[j]
      ENDIF
      // increment the pointer pointing to the item in the list
      k = k + 1
    ENDWHILE

    WHILE i < leftHalf.LENGTH
      mergeList[k] = leftHalf[i]
      // increment the pointer pointing to the item in the leftHalf
      i = i + 1
      // increment the pointer pointing to the item in the list
      k = k + 1
    ENDWHILE

    WHILE j < rightHalf.LENGTH
      mergeList[k] = rightHalf[j]
      // increment the pointer pointing to the item in the rightHalf
      j = j + 1
      // increment the pointer pointing to the item in the list
      k = k + 1
    ENDWHILE
  ENDIF
ENDPROCEDURE
```

SORTING ALGORITHMS

QUICK SORT

Quick sort is a divide and conquer sorting algorithm where a pivot value is used, such as the first item in the list. The remainder of the list is divided into two partitions where: all elements less than the pivot value must be in the first partition; and all elements greater than the pivot value must be in the second partition. The implementation of a quick sort is usually recursive as the way it solves the problem is inherently recursive so it naturally lends itself to this style of implementation.

Process

- 1) Select a pivot value, sometimes the first item in the list.
- 2) Locate the two position markers:
 - set leftMark variable to the index of the second item in the list (after the pivot); and
 - set rightMark variable to the index of the last item in the list.
- 3) Move the items which are less than or equal to the pivot to the left-hand side of the pivot and move the items which are greater than the pivot to the right-hand side of the pivot:
 - compare the pivot to the item at the leftMark and if the item at the leftMark is less than pivot, increment the leftMark (move to the right) – repeat this until the pivot is greater than the item at the leftMark.
 - compare the pivot to the item at the rightMark and if the item at the rightMark is greater than the pivot, decrement the rightMark (move to the left) – repeat this until the pivot is less than the item at the rightMark.
- 4) Exchange the item at leftMark with the item at rightMark and repeat stage 3).
- 5) When $\text{rightMark} < \text{leftMark}$, the split point is at the position of the rightMark.
- 6) Exchange the item at the pivot with the item at the split point.
- 7) Divide the list at the split point and recursively quick sort each half.

Pseudocode

```

FUNCTION partition(list, start, end)
    pivot = list[start]    // set the pivot to point to the first item
    leftMark = start + 1  // set the leftMark to point to the item after the pivot
    rightMark = end      // set the rightMark to point to the last item
    done = False        // the split point has not been found
    WHILE done == False  // while the split point has not been found
        // while the leftMark is less than or equal to the rightMark and the leftMark
        // is less than or equal to the pivot
        WHILE leftMark <= rightMark AND list[leftMark] <= pivot
            // increment the leftMark
            leftMark = leftMark + 1
        ENDWHILE
        // while the rightMark is greater than or equal to the leftMark and the
        // rightMark is greater than or equal to the pivot
        WHILE rightMark >= leftMark AND list[rightMark] >= pivot
            // decrement the rightMark
            rightMark = rightMark - 1
        ENDWHILE
        // if the pointer have swapped over
        IF rightMark < leftMark THEN
            // the split point has been found
            done = True
        ELSE
            // the split point has not been found so swap the items at the leftMark and
            // the rightMark
            temp = list[leftMark]
            list[leftMark] = list[rightMark]
            list[rightMark] = temp
        ENDIF
    ENDWHILE
    // swap the item at the pivot with the item at the rightMark
    temp = list[start]
    list[start] = list[rightMark]
    list[rightMark] = temp
    // return the split point (rightMark)
    RETURN rightMark
ENDFUNCTION

FUNCTION quicksort(list, start, end)
    // base case
    IF start < end THEN
        // partition the list
        split = partition(list, start, end)
        // recursive case - quick sort the right half
        quickSort(list, start, split-1)
        // recursive case - quick sort the left half
        quickSort(list, split+1, end)
    ENDIF
    RETURN list
ENDFUNCTION

list = [9, 5, 4, 15, 3, 8, 11]
sortedList = quicksort(list, 0, list.LENGTH-1)
PRINT(sortedList)

```


SORTING ALGORITHMS

COMPARISON

		Bubble Sort	Insertion Sort	Merge Sort	Quick Sort
Time Complexity	Best Case	Linear $O(n)$	Linear $O(n)$	Logarithmic $O(n \log n)$	Logarithmic $O(n \log n)$
	Average Case	Polynomial $O(n^2)$	Polynomial $O(n^2)$	Logarithmic $O(n \log n)$	Logarithmic $O(n \log n)$
	Worst Case	Polynomial $O(n^2)$	Polynomial $O(n^2)$	Polynomial $O(n \log n)$	Polynomial $O(n^2)$
Space Complexity (Auxiliary Worst Case)		Constant $O(1)$	Constant $O(1)$	Linear $O(n)$	Logarithmic $O(\log n)$
Avoiding the worst case time		Sort the list from both directions. Decrement the number of items to be inspected on each pass, as these missed items are assumed to be sorted.	-	Cannot be optimised because it takes $O(\log n)$ to break the array down into the sub lists and then $O(n)$ swaps are made.	Choosing an appropriate pivot value; a common method is to use the median of the leftmark rightmark and middle value of the array as the pivot value. This is recalculated on each recursive call.
Coding difficulty (1 – easiest, 4 – hardest)		1	2	3	4
Evaluation		Slowest but easiest to code.	Polynomial time complexity but reduced to linear if list is almost sorted. Worst case if data is in descending order.	Scales well since logarithmic but requires additional memory space for the merging process. Recursion could lead to a stack overflow if the list is very large.	Generally fastest but dependent on using a pivot which is not close to the smallest or largest elements of the list. Rarely worst case especially if pivot value has been chosen carefully. Does not required additional memory space, operations are completed “in place”.
		Number of comparisons is the same. Less swaps made in insertion, thus less writing.			In a partially or fully sorted list, bubble or insertion may actually be better than merge sort and they are simpler to code.

Linear search is a brute force and iterative searching algorithm which sequentially checks each element of the list to see if it matches the search criteria until a match is found or until all the elements have been searched.

Process

- 1) Set found variable to False.
- 2) Set index variable to 0.
- 3) While the item has not been found and the index is within range of the list:
 - if the item at the current value of index is equal to the search criteria, set the found variable to True and return the item at the current value of index;
 - else, increment index.
- 4) If the item is not found, the index will become greater than the length of the list and the while loop will finish, this means that the item has not been found.

Pseudocode

```
FUNCTION linearSearch(list, searchCriteria)
  found = False
  index = 0
  WHILE found = False AND index < list.LENGTH
    IF list[index] == searchCriteria THEN
      found = True
      RETURN list[index]
    ELSE
      index = index + 1
    ENDIF
  ENDWHILE
  RETURN "Item not found"
ENDFUNCTION
```

Binary search is a divide and conquer iterative searching algorithm which works by repeatedly dividing in half the portion of a list which contains the required data item until there is only one item in the list. This can also be implemented recursively.

Process

- 1) Set found variable to False.
- 2) Set lowerBound variable to 0.
- 3) Set upperBound variable to list.LENGTH-1 (index of the last item).
- 4) While the item has not been found and the lowerBound is less than or equal to the upperBound:
 - calculate a midpoint by doing floor division of the sum of lowerBound and upperBound;
 - if the item at the midpoint is equal to the search criteria, set found to True;
 - if the item at the midpoint is less than the search criteria, set the lowerBound to midpoint + 1;
 - if the item at the midpoint is greater than the search criteria, set the upperBound to midpoint - 1.
- 5) Return the found variable.

Pseudocode

```
FUNCTION binarySearch(list, searchCriteria)
    found = False
    lowerBound = 0
    upperBound = list.LENGTH-1
    WHILE found == False AND lowerBound <= upperBound
        midPoint = (lowerBound + upperBound) DIV 2
        IF list[midPoint] == searchCriteria THEN
            found = True
        ELIF list[midPoint] < searchCriteria THEN
            lowerBound = midPoint + 1
        ELSE
            upperBound = midPoint - 1
    ENDWHILE
    RETURN found
ENDFUNCTION
```

SEARCHING ALGORITHMS

COMPARISON

		Linear Search	Binary Search
Time Complexity	Best Case	Constant $O(1)$	Constant $O(1)$
	Average Case	Linear $O(n)$	Logarithmic $O(\log n)$
	Worst Case	Linear $O(n)$	Logarithmic $O(\log n)$
Space Complexity (Auxiliary Worst Case)		Constant $O(1)$	Constant $O(1)$
Avoiding the worst case time		Cannot be optimised.	Can be implemented recursively. This could be considered the optimum implementation as it is naturally recursive.
Coding difficulty (1 – easiest, 4 – hardest)		1	2
Evaluation		<p>Works on any data set.</p> <p>If the item is the last in the list, then it will take $O(n)$.</p> <p>More suited to smaller data sets.</p>	<p>The data set must be in order.</p> <p>Very efficient even if list is large.</p> <p>If data isn't sorted, consider the complexity of a sorting algorithm.</p>